

Resonance form factors in lattice QCD

Andria Agadjanov, University of Bonn

Brookhaven, February 5, 2015

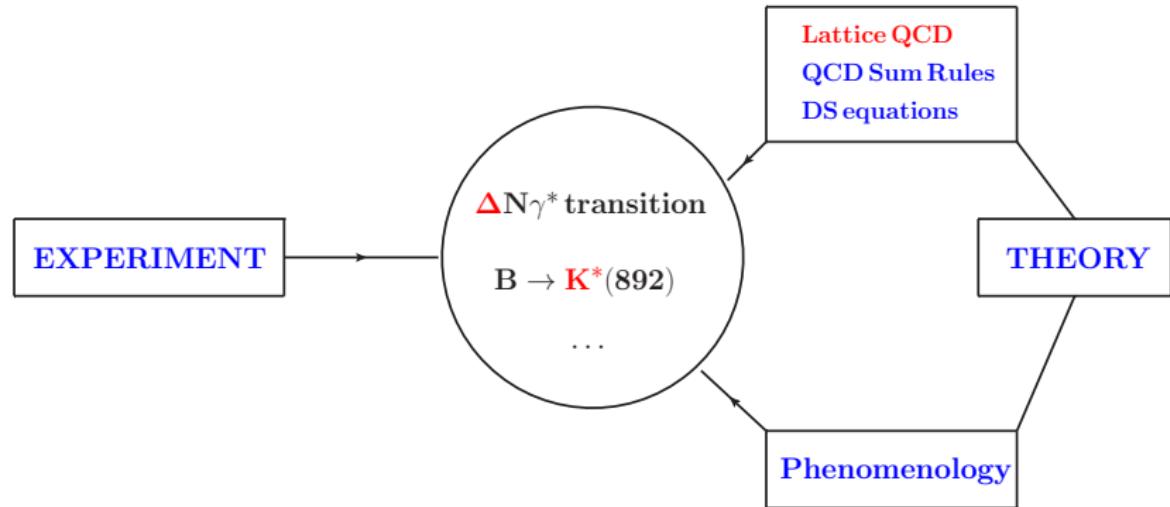
AA, V. Bernard, U.-G. Meißner, A. Rusetsky, Nucl. Phys. B 886 (2014)



Outline

- ▶ Introduction: the $\Delta N\gamma^*$ transition
- ▶ Motivation
- ▶ Goals, results
- ▶ Outlook: the $B \rightarrow K^*$

Introduction

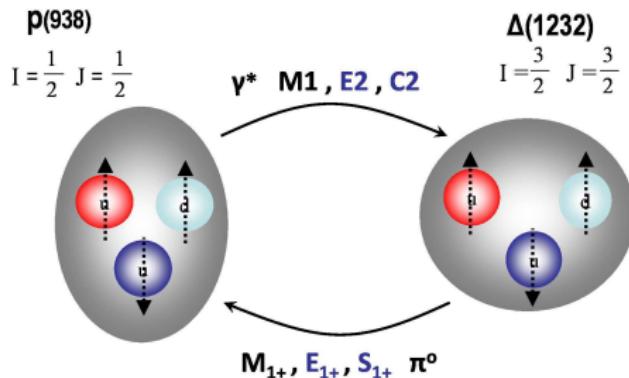


- $\gamma^* N \rightarrow \pi N$ near the $\Delta(1232)$: study of the **hadron deformation**
- $B \rightarrow K^*\gamma^*$, $B \rightarrow K^*l^+l^-$ with $K^*(892) \rightarrow K\pi$: sensitive to **NP**

$$\gamma^* N \rightarrow \Delta$$

$$\begin{aligned}\gamma^* p &\rightarrow \Delta^+(1232) \rightarrow p\pi^0 \quad (66\%) \\ \gamma^* p &\rightarrow \Delta^+(1232) \rightarrow n\pi^+ \quad (33\%) \\ \gamma^* p &\rightarrow \Delta^+(1232) \rightarrow p\gamma \quad (0.56\%)\end{aligned}$$

↪ study of the de-excitation radiation pattern



Spherical $\Rightarrow M1$

Deformed $\Rightarrow M1, E2, C2$

- The electromagnetic transition matrix element:

$$\langle \Delta(P, \lambda) | J_\mu(0) | N(Q, \epsilon) \rangle = \left(\frac{2}{3}\right)^{1/2} \bar{u}^\sigma(P, \lambda) \mathcal{O}_{\sigma\mu} u(Q, \epsilon),$$

with the Lorentz-structure

$$\mathcal{O}_{\sigma\mu} = G_M(Q^2) K_{\sigma\mu}^{M1} + G_E(Q^2) K_{\sigma\mu}^{E2} + G_C(Q^2) K_{\sigma\mu}^{C2}$$

H. F. Jones and M. D. Scadron, Ann. Phys. **81** (1973) 1

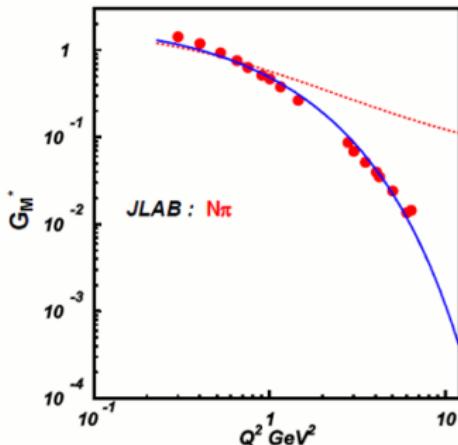
$$\text{EMR} \equiv \frac{\text{Im} E_{1+}^{3/2}}{\text{Im} M_{1+}^{3/2}} = -\frac{G_E(Q^2)}{G_M(Q^2)}, \quad \text{CMR} \equiv \frac{\text{Im} L_{1+}^{3/2}}{\text{Im} M_{1+}^{3/2}} = -\frac{|\vec{Q}|}{2m_\Delta} \frac{G_C(Q^2)}{G_M(Q^2)}$$

↪ extracted from experiment calculated on the lattice ↪

Experiment vs. theory

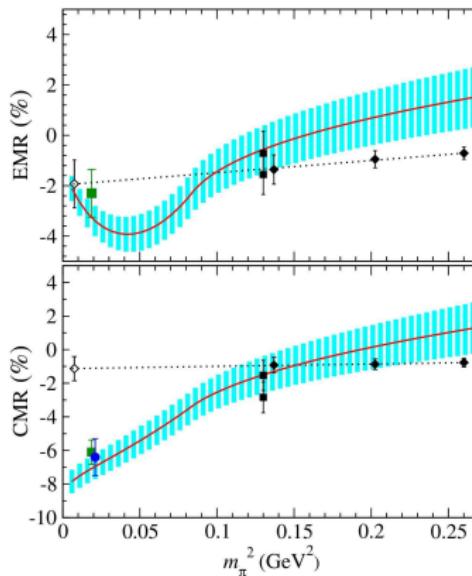
... notwithstanding its steady progress toward results with input parameters that approximate the real world, the numerical simulation of lattice-regularised QCD will not suffice. Approaches formulated in the continuum and inspired by, based upon, or connected directly with QCD are necessary.

C. D. Roberts arXiv:1501.06581 (2015)



- $\gamma^* N \rightarrow \Delta$: infrared evolution of QCDs β -function

The values of EMR and CMR



χ EFT extrapolation of lattice results to the physical pion mass

V. Pascalutsa, M. Vanderhaeghen, Phys. Rev. D 73 (2006) 034003

- The Δ is treated as a **stable** particle!

Motivation

The largest conceptual question as we enter the chiral regime in full QCD, is how to fully incorporate the physical effect of the decay of the Δ into a pion and nucleon on the transition form factors.

C. Alexandrou et al., Phys. Rev. D **77** (2008) 085012

→ seminal work on $K \rightarrow \pi\pi$ by

L. Lellouch and M. Lüscher, Commun.Math.Phys. **219**, 31 (2001)

- generalization of Lellouch-Lüscher method for matrix elements, involving **resonances** is needed

The framework: non-relativistic EFT

- Basic properties of the theory:

- ▷ the total number of heavy particles is conserved
- ▷ manifestly Lorentz-invariant formulation is possible
- ▷ the theory is matched to the full QFT (e.g., ChPT).



Bubble-chain diagrams

$$T \propto 1 + cJ + c^2 J^2 + \dots = \frac{1}{1 - cJ}$$

- A bridge: finite volume spectrum \leftrightarrow scattering sector

G. Colangelo, J. Gasser, B. Kubis, A. Rusetsky, Phys. Lett. B **638** (2006) 187
J. Gasser, B. Kubis, A. Rusetsky, Nucl. Phys. B **850** (2011) 96

Goals, results

- Previous work:
 - D. Hoja, U.-G. Meißner, A. Rusetsky, JHEP 1004 (2010) 050
 - V. Bernard, D. Hoja, U.-G. Meißner, A. Rusetsky, JHEP 1209 (2012) 023
- scalar resonance form factor in the external scalar field (analog: $\Delta\Delta\gamma^*$)
- The $\Delta N\gamma^*$ transition:
 - ▷ inclusion of *spin*;
 - ▷ generalization to *transition* form factors.

Extraction of the form factors

- on the **real** energy axis → model-dependent

I. G. Aznauryan, V. D. Burkert and T. -S. H. Lee, arXiv:0810.099
D. Drechsel, O. Hanstein, S.S. Kamalov, L. Tiator, Nucl. Phys. A 645 (1999) 145

- at the resonance **pole** → *process-independent*

▷ **definition** of the resonance matrix element:

$$\langle P, \text{resonance} | J(0) | Q, \text{stable} \rangle = \lim_{P^2 \rightarrow s_R, Q^2 \rightarrow M^2} Z_R^{-1/2} Z^{-1/2} (s_R - P^2) (M^2 - Q^2) F(P, Q)$$

↪ **analytic continuation** to the pole is unavoidable

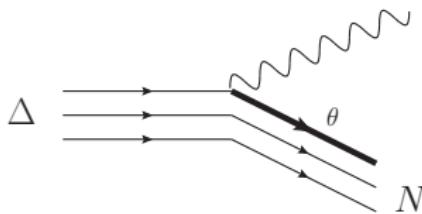
S. Mandelstam, Proc. Roy. Soc. Lond. A 233 (1955) 248.

- infinitely narrow resonances → identical results

↪ however, discrepancies in the results could be sizable:

R. L. Workman, L. Tiator and A. Sarantsev, Phys. Rev. C 87 (2013) 6, 068201

Kinematics



- The Δ is at rest $\mathbf{P} = 0$ and nucleon momentum \mathbf{Q} along 3-axis.
- To perform the fit (*see below*): vary p , while $|\mathbf{Q}|$ fixed.
 - ▷ (partially) twisted boundary conditions;
 - ↪ nucleon form factors: M. Göckeler et al., PoS LATTICE 2008 (2008) 138
 - ▷ asymmetric boxes $L \times L \times L'$.Alexandru, Döring
- Note: in practice, the case $\mathbf{P} \neq 0$ could be preferable.

Spin

- Spin-projecting local operators:

$$G_2 \quad \mathcal{O}_{3/2}(t) = \sum_{\mathbf{x}} \frac{1}{2} (1 + \Sigma_3) \frac{1}{2} (1 + \gamma_4) \frac{1}{\sqrt{2}} (\mathcal{O}^1(\mathbf{x}, t) - i \Sigma_3 \mathcal{O}^2(\mathbf{x}, t)),$$

$$G_2 \quad \mathcal{O}_{1/2}(t) = \sum_{\mathbf{x}} \frac{1}{2} (1 - \Sigma_3) \frac{1}{2} (1 + \gamma_4) \frac{1}{\sqrt{2}} (\mathcal{O}^1(\mathbf{x}, t) + i \Sigma_3 \mathcal{O}^2(\mathbf{x}, t)),$$

$$G_1 \quad \tilde{\mathcal{O}}_{1/2}(t) = \sum_{\mathbf{x}} \frac{1}{2} (1 + \Sigma_3) \frac{1}{2} (1 + \gamma_4) \mathcal{O}^3(\mathbf{x}, t),$$

$$\bar{\psi}_{\pm 1/2}^{\mathbf{Q}}(t) = \sum_{\mathbf{x}} e^{i \mathbf{Q} \cdot \mathbf{x}} \bar{\psi}(\mathbf{x}, t) \frac{1}{2} (1 \pm \Sigma_3) \frac{1}{2} (1 + \gamma_4), \quad \Sigma_3 = \text{diag}(\sigma_3, \sigma_3)$$

- Irreps G_1, G_2 : no $S - P$ -wave mixing (the P_{31} wave is small)

$$p \cot \delta(\mathbf{p}) = \frac{2}{\sqrt{\pi} L} \left\{ \hat{Z}_{00}(1; q^2) \pm \frac{1}{\sqrt{5} q^2} \hat{Z}_{20}(1; q^2) \right\} \equiv -p \cot \phi(q), \quad q = \frac{pL}{2\pi}$$

↪ Lüscher eq. for the P_{33} wave: M. Göckeler, Phys. Rev. D **86** (2012) 094513

Lattice correlators

- The two-point functions:

$$\tilde{D}_{1/2}(t) = \text{Tr} \langle 0 | \tilde{\mathcal{O}}_{1/2}(t) \bar{\tilde{\mathcal{O}}}_{1/2}(0) | 0 \rangle,$$

$$D_{1/2}(t) = \text{Tr} \langle 0 | \mathcal{O}_{1/2}(t) \bar{\mathcal{O}}_{1/2}(0) | 0 \rangle,$$

$$D_{3/2}(t) = \text{Tr} \langle 0 | \mathcal{O}_{3/2}(t) \bar{\mathcal{O}}_{3/2}(0) | 0 \rangle,$$

$$D_{\mathbf{Q}}^{\pm}(t) = \text{Tr} \langle 0 | \psi_{\pm 1/2}^{\mathbf{Q}}(t) \bar{\psi}_{\pm 1/2}^{\mathbf{Q}}(0) | 0 \rangle.$$

- The three-point functions:

$$\tilde{R}_{1/2}(t', t) = \langle 0 | \tilde{\mathcal{O}}_{1/2}(t') J^3(0) \bar{\psi}_{1/2}^{\mathbf{Q}}(t) | 0 \rangle,$$

$$R_{1/2}(t', t) = \langle 0 | \mathcal{O}_{1/2}(t') J^+(0) \bar{\psi}_{-1/2}^{\mathbf{Q}}(t) | 0 \rangle,$$

$$R_{3/2}(t', t) = \langle 0 | \mathcal{O}_{3/2}(t') J^+(0) \bar{\psi}_{1/2}^{\mathbf{Q}}(t) | 0 \rangle,$$

where

$$J^+(0) = \frac{1}{\sqrt{2}} (J^1(0) + iJ^2(0)).$$



- In the limit $t' \rightarrow +\infty, t \rightarrow -\infty$:

$$\begin{aligned} \mathcal{N} \frac{\text{Tr}(\tilde{R}_{1/2}(t', t))}{\tilde{D}_{1/2}(t' - t)} \left(\frac{D_{\mathbf{Q}}^+(t') \tilde{D}_{1/2}(-t) \tilde{D}_{1/2}(t' - t)}{\tilde{D}_{1/2}(t') D_{\mathbf{Q}}^+(-t) D_{\mathbf{Q}}^+(t' - t)} \right)^{1/2} &\rightarrow \langle 1/2 | J^3(0) | 1/2 \rangle, \\ -\mathcal{N} \frac{\text{Tr}(R_{1/2}(t', t))}{D_{1/2}(t' - t)} \left(\frac{D_{\mathbf{Q}}^-(t') D_{1/2}(-t) D_{1/2}(t' - t)}{D_{1/2}(t') D_{\mathbf{Q}}^-(-t) D_{\mathbf{Q}}^-(t' - t)} \right)^{1/2} &\rightarrow \langle 1/2 | J^+(0) | -1/2 \rangle, \\ -\mathcal{N} \frac{\text{Tr}(R_{3/2}(t', t))}{D_{3/2}(t' - t)} \left(\frac{D_{\mathbf{Q}}^+(t') D_{3/2}(-t) D_{3/2}(t' - t)}{D_{3/2}(t') D_{\mathbf{Q}}^+(-t) D_{\mathbf{Q}}^+(t' - t)} \right)^{1/2} &\rightarrow \langle 3/2 | J^+(0) | 1/2 \rangle, \end{aligned}$$

where $\mathcal{N} = (4E\sqrt{m_N^2 + \mathbf{Q}^2})^{1/2}$.

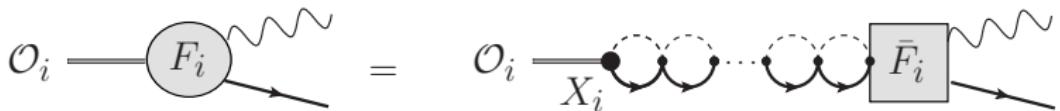
||

$F_i, i=1, 2, 3$

\Rightarrow three $\Delta N\gamma^*$ form factors are **separately** projected out.

\hookrightarrow similar procedure can be applied to fields with other spin

$\Delta N\gamma^*$ vertex



- ▷ The $F_i = F_i(p_n, |\mathbf{Q}|)$, $i=1, 2, 3 \rightarrow G_M, G_E, G_C$ form factors, are measured on *the lattice*.
- ▷ The $\bar{F}_i(p_n, |\mathbf{Q}|)$ are *volume-independent* irreducible amplitudes.

$$|\bar{F}_i(p_n, |\mathbf{Q}|)| = \underbrace{\sqrt{V}^{1/2} \left(\frac{\cos^2 \delta(p_n)}{|\delta'(p_n) + \phi'(q_n)|} \frac{p_n^2}{2\pi} \right)^{-1/2}}_{\text{LL factor}} |F_i(p_n, |\mathbf{Q}|)|$$

Real axis

- The \bar{F}_i are related to the $\gamma^* N \rightarrow \pi N$ multipole amplitudes

$$\mathcal{A}_i(p, |\mathbf{Q}|) = e^{i\delta(p)} \cos \delta(p) \bar{F}_i(p, |\mathbf{Q}|)$$

↔ Watson's theorem

- $\mathcal{A}_i(p_A, |\mathbf{Q}|) = 0$ at $\delta(p_A) = \pi/2$? **No!**

$$\mathcal{A}_i(p, |\mathbf{Q}|) = \underbrace{\frac{e^{i\delta(p)}}{p^3} \sin \delta(p)}_{\text{potential}} \underbrace{p^3 \cot \delta(p)}_{\text{zero at } p^2 = p_A^2} \underbrace{\bar{F}_i(p, |\mathbf{Q}|)}_{\text{diverges at } p^2 = p_A^2}$$

$$\mathcal{A}_i(p_n, |\mathbf{Q}|) = e^{i\delta(p_n)} V^{1/2} \left(\frac{1}{|\delta'(p_n) + \phi'(q_n)|} \frac{p_n^2}{2\pi} \right)^{-1/2} |\bar{F}_i(p_n, |\mathbf{Q}|)|$$

↔ **LL equation** for the photoproduction amplitude in the elastic region

- The $\mathcal{A}_i \leftrightarrow$ multipoles:

$$\begin{aligned}\tilde{\mathcal{A}}_{1/2} &= -16\pi i E \sqrt{2} \frac{\sqrt{Q^2}}{|\mathbf{Q}|} S_{1+}, \\ \mathcal{A}_{1/2} &= -\frac{1}{2}(3E_{1+} + M_{1+})(-16\pi i E), \\ \mathcal{A}_{3/2} &= \frac{\sqrt{3}}{2}(E_{1+} - M_{1+})(-16\pi i E).\end{aligned}$$

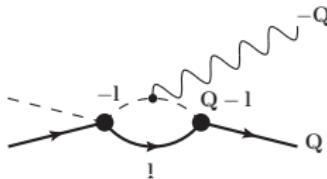
- The narrow width approximation:

$$|\text{Im } \mathcal{A}_i(p_A, |\mathbf{Q}|)| = \sqrt{\frac{8\pi}{p_A \Gamma}} |F_i^A(p_A, |\mathbf{Q}|)|,$$

$p = p_A$ - Breit-Wigner pole, $F_i^A(p_A, |\mathbf{Q}|) \rightarrow \Delta N\gamma^*$ form factors.

I. G. Aznauryan, V. D. Burkert and T. -S. H. Lee, arXiv:0810.099

Analytic continuation



A potentially dangerous diagram in the \bar{F}_i

$$I = \frac{1}{V} \sum_{\mathbf{l}} \frac{1}{8w_1(\mathbf{l})w_2(-\mathbf{l})w_2(\mathbf{Q}-\mathbf{l})} \frac{1}{(w_1(\mathbf{l}) + w_2(-\mathbf{l}) - E_n)(w_1(\mathbf{l}) + w_2(\mathbf{Q}-\mathbf{l}) - Q^0)}$$

- After some transformations \Rightarrow

$$I = \underbrace{\frac{1}{V} \sum_{\mathbf{l}} \frac{1}{2E_n(\mathbf{l}^2 - p_n^2)}}_{\propto p_n \cot \delta(p_n)} \underbrace{\frac{1}{2} \int_{-1}^1 dy \frac{1}{2\hat{w}_2(\hat{w}_1 + \hat{w}_2 - Q^0)}}_{\text{polynomial in } p_n^2}$$

$$\hat{w}_1 = \sqrt{m_N^2 + p_n^2}, \quad \hat{w}_2 = \sqrt{M_\pi^2 + p_n^2 + \mathbf{Q}^2 - 2|\mathbf{Q}|p_n y}.$$

$\Rightarrow p^2 I$ is a low-energy **polynomial** in p^2

Complex plane

- The $\Delta N\gamma^*$ matrix elements, evaluated at **the pole** $p = p_R$:

$$F_i^R(p_R, |\mathbf{Q}|) = Z_R^{1/2} \bar{F}_i(p_R, |\mathbf{Q}|),$$

$$Z_R = \left(\frac{p_R}{8\pi E_R} \right)^2 \left(\frac{16\pi p_R^3 E_R^3}{w_{1R} w_{2R} (2p_R h'(p_R^2) + 3ip_R^2)} \right), \quad w_{iR} = \sqrt{m_i^2 + p_R^2},$$

$$p^3 \cot \delta(p) \doteq h(p^2) = -\frac{1}{a} + \frac{1}{2} rp^2 + \dots, \quad h(p_R^2) = -ip_R^3$$

- ▷ The narrow width approximation ($p_R \rightarrow p_n$):

$$F_i^R(p_R, |\mathbf{Q}|) \rightarrow F_i^A(p_A, |\mathbf{Q}|) \quad \text{as } p_R \rightarrow p_A !$$

$$F_i^R(p_n, |\mathbf{Q}|) = V^{1/2} \left(\frac{E_n}{2w_{1n} w_{2n}} \right)^{1/2} F_i(p_n, |\mathbf{Q}|)$$

↪ proper normalization of states

Prescription on the lattice

- Measure the $F_i(p, |\mathbf{Q}|)$ at different values of p with $|\mathbf{Q}|$ fixed.
- Real energy: extract the multipole amplitudes (*see above*).
- Extraction of the matrix elements at the resonance pole:
 1. fit the functions $p^3 \cot \delta(p) \bar{F}_i(p, |\mathbf{Q}|)$

$$p^3 \cot \delta(p) \bar{F}_i(p, |\mathbf{Q}|) = A_i(|\mathbf{Q}|) + p^2 B_i(|\mathbf{Q}|) + \dots$$

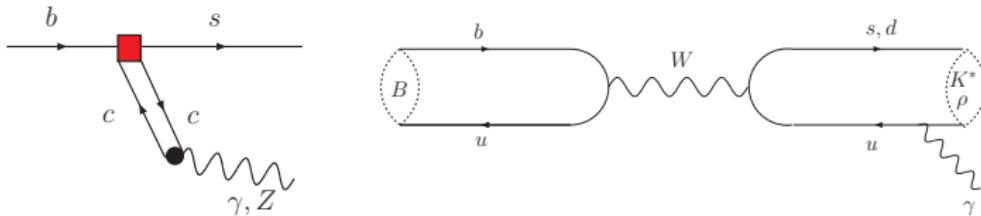
2. evaluate the resonance matrix elements by substitution

$$F_i^R(p_R, |\mathbf{Q}|) = i p_R^{-3} Z_R^{1/2} (A_i(|\mathbf{Q}|) + p_R^2 B_i(|\mathbf{Q}|) + \dots).$$

- No finite fixed points \Rightarrow easier to measure than $\Delta\Delta\gamma^*$ vertex

$B \rightarrow K^*$ transitions

- A problem: long distance contributions.



- Recent lattice measurement:

R. R. Horgan, Z. Liu, S. Meinel, M. Wingate, Phys. Rev. D **89**, 094501 (2014)

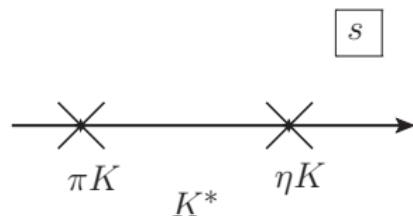
↪ consistent with LCSR determinations

- Related work: multichannel LL equation, kinematics.

R. A. Briceño, M.T. Hansen, A. Walker-Loud, arXiv:1406.5965 (2014)

Outlook

- ▶ Rigorous physical analysis of the problem.
- ▶ Extraction of the matrix elements at the pole: two-channel case.



Thank you!

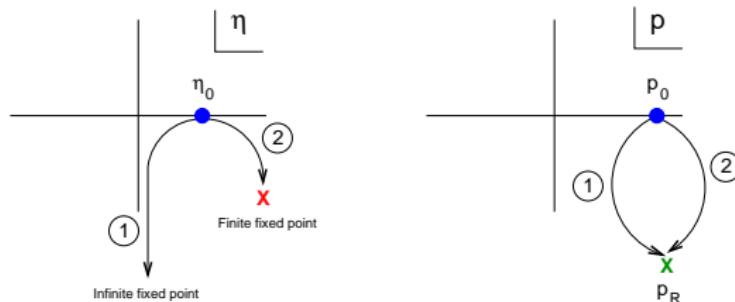
Finite fixed points ($\Delta\Delta\gamma^*, \dots$)

- The corresponding integral I : V. Bernard, et. al., JHEP 1209 (2012)

023

$$I = I_\infty + \frac{1}{32\pi E p} (1 + \cot^2 \delta(p)) \eta \phi'(\eta) \Bigg|_{p=p_n}, \quad \eta = \frac{pL}{2\pi}$$

- $p \rightarrow p_R$: $Z_{00}(1; \eta_R^2) + i\pi^{3/2}\eta_R = 0 \rightarrow \exists \eta_R : |\text{Im } \eta_R| < \infty$



- Now, $\cot \phi(\eta) + i \propto \eta - \eta_R$ and $\phi(\eta) \propto \ln(\eta - \eta_R) \Rightarrow$

$$(\cot \phi(\eta) + i)\phi'(\eta) \rightarrow \text{const} \neq 0$$

- Solution: perform the measurement at two energy levels